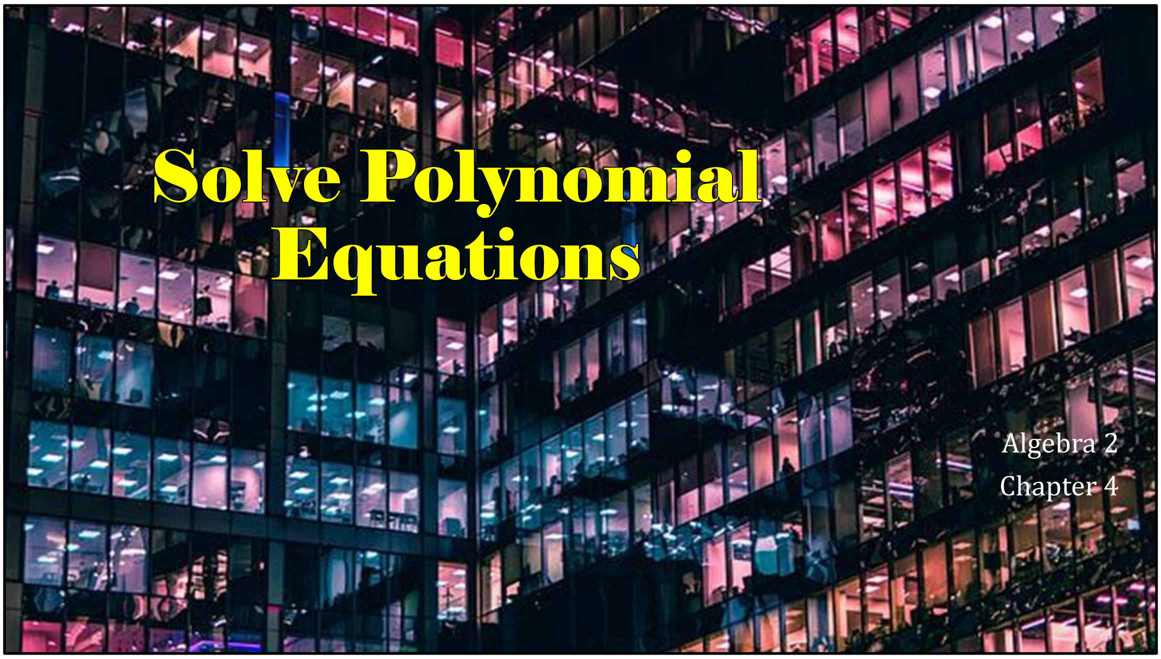
Solve Polynomial Equations
Algebra 2
Chapter 4

- This Slideshow was developed to accompany the textbook
  - *Big Ideas Algebra 2*
  - *By Larson, R., Boswell*
  - *2022 K12 (National Geographic/Cengage)*
- Some examples and diagrams are taken from the textbook.

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4-01 Add, Subtract, and Multiply Polynomials (4.2)

Objectives:

- Add and subtract polynomial expressions.
- Multiply polynomial expressions.

4-01 Add, Subtract, and Multiply Polynomials (4.2)

- Adding, subtracting, and multiplying are always good things to know how to do.
- Sometimes you might want to combine two or more models into one big model.

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4-01 Add, Subtract, and Multiply Polynomials (4.2)

- **Adding and subtracting polynomials**
  - Add or subtract the coefficients of the terms with the same power.
  - Called *combining like terms*.
- Simplify
  - $(5x^2 + x - 7) + (-3x^2 - 6x - 1)$
  - $(3x^3 + 8x^2 - x - 5) - (5x^3 - x^2 + 17)$

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$2x^2 - 5x - 8$

$-2x^3 + 9x^2 - x - 22$

4-01 Add, Subtract, and Multiply Polynomials (4.2)

- **Multiplying polynomials**
  - Use the distributive property
- Simplify
  - $(x - 3)(x + 4)$
- $(x + 2)(x^2 + 3x - 4)$

$x \cdot x + 4x - 3x - 12 \rightarrow x^2 + x - 12$

$x \cdot x^2 + x \cdot 3x - x \cdot 4 + 2 \cdot x^2 + 2 \cdot 3x - 2 \cdot 4 \rightarrow x^3 + 3x^2 - 4x + 2x^2 + 6x - 8 \rightarrow x^3 + 5x^2 + 2x - 8$

4-01 Add, Subtract, and Multiply Polynomials (4.2)

- $(x - 1)(x + 2)(x + 3)$

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$(x^2 + 2x - 1x - 2)(x + 3) \rightarrow (x^2 + x - 2)(x + 3) \rightarrow x^2(x + 3) + x(x + 3) - 2(x + 3) \rightarrow x^3 + 3x^2 + x^2 + 3x - 2x - 6 \rightarrow x^3 + 4x^2 + x - 6$

4-01 Add, Subtract, and Multiply Polynomials (4.2)

- **Special Product Patterns**
  - Sum and Difference
    - $(a - b)(a + b) = a^2 - b^2$
  - Square of a Binomial
    - $(a \pm b)^2 = a^2 \pm 2ab + b^2$
  - Cube of a Binomial
    - $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

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4-01 Add, Subtract, and Multiply Polynomials (4.2)

- $(x + 2)^2$
- $(x - 3)^2$

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$(x + 2)(x + 2) \rightarrow x^2 + 2(2x) + 2^2 \rightarrow x^2 + 4x + 4$

$x^2 + 2(-3x) + (-3)^2 \rightarrow x^2 - 6x + 9$

4-02 Factor and Solve Polynomial Equations (4.4)

Objectives:
- Factor polynomial expressions.
- Solve polynomial equations by factoring.

4-02 Factor and Solve Polynomial Equations (4.4)

- A manufacturer of shipping cartons who needs to make cartons for a specific use often has to use special relationships between the length, width, height, and volume to find the exact dimensions of the carton.
- The dimensions can usually be found by writing and solving a polynomial equation.
- This lesson looks at how factoring can be used to solve such equations.

4-02 Factor and Solve Polynomial Equations (4.4)

- **<u>How to Factor</u>**

1. **Greatest Common Factor**
   - Comes from the distributive property
   - If the same number or variable is in each of the terms, you can bring the number to the front times everything that is left.
   - $3x^2y + 6xy - 9xy^2 =$
   - Look for this first!

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$3xy(x + 2 - 3y)$

4-02 Factor and Solve Polynomial Equations (4.4)

2. Check to see how many terms

- *Two terms (formulas)*
  - Difference of two squares: $a^2 - b^2 = (a - b)(a + b)$
    - $9x^2 - y^4 =$
  - Sum of Two Cubes: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
    - $8x^3 + 27 =$
  - Difference of Two Cubes: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
    - $y^3 - 8 =$

$(3x - y^2)(3x + y^2)$

$(2x + 3)(4x^2 - 6x + 9)$

$(y - 2)(y^2 + 2y + 4)$

4-02 Factor and Solve Polynomial Equations (4.4)

- *Three terms (General Trinomials → $ax^2 + bx + c$)*
  - Write two sets of parentheses (   )(   )
  - Guess and Check
    - The Firsts multiply to make ax^2
    - The Lasts multiply to make c
    - The Outers + Inners make bx
  - $x^2 + 7x + 10 =$
  - $6x^2 - 7x - 20 =$

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(x+2)(x+5)

(2x – 5)(3x + 4)

4-02 Factor and Solve Polynomial Equations (4.4)

- *Four terms (Grouping)*
    - Group the terms into sets of two so that you can factor a common factor out of each set
    - Then factor the factored sets (Factor twice)
  - $b^3 - 3b^2 - 4b + 12 =$

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$(b^3 - 3b^2) + (-4b + 12) = b^2(b - 3) + -4(b - 3) = (b - 3)(b^2 - 4) = (b - 3)(b - 2)(b + 2)$

4-02 Factor and Solve Polynomial Equations (4.4)

3. Try factoring more!

- $a^2x - b^2x + a^2y - b^2y =$

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$x(a^2 - b^2) + y(a^2 - b^2) = (x + y)(a^2 - b^2) = (x + y)(a - b)(a + b)$

4-02 Factor and Solve Polynomial Equations (4.4)

- $3a^2z - 27z =$
- $n^4 - 81 =$

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$3z(a^2 - 9) = 3z(a - 3)(a + 3)$

$(n^2 - 9)(n^2 + 9) = (n^2 + 9)(n - 3)(n + 3)$

4-02 Factor and Solve Polynomial Equations (4.4)

- **Solving Equations by Factoring**
  1. Make = 0
  2. Factor
  3. Make each factor = 0 because if one factor is zero, 0 time anything = 0

4-02 Factor and Solve Polynomial Equations (4.4)

- $2x^5 = 18x$

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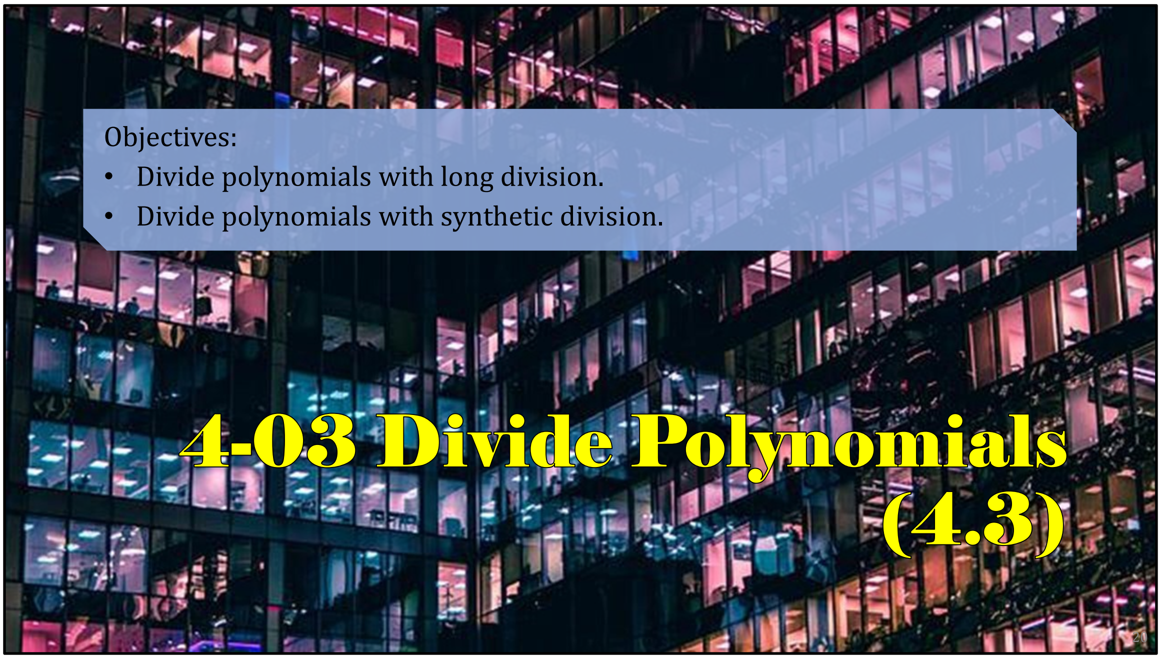
$$2x^5 - 18x = 0$$
$$2x(x^4 - 9) = 0$$
$$2x(x^2 - 3)(x^2 + 3) = 0$$
$$2x = 0, x^2 - 3 = 0, x^2 + 3 = 0$$
$$x = 0, \pm\sqrt{3}, \pm\sqrt{3}i$$


Objectives:
• Divide polynomials with long division.
• Divide polynomials with synthetic division.
4-03 Divide Polynomials (4.3)

4-03 Divide Polynomials (4.3)

- So far we done add, subtracting, and multiplying polynomials.
- Factoring is similar to division, but it isn't really division.
- Today we will deal with real polynomial division.

4-03 Divide Polynomials (4.3)

- **Polynomial Long Division**

1. Set up the division problem. $\boldsymbol{divisor)\overline{dividend}}$
2. **Divide** the leading term of the dividend by the leading term of the divisor.
3. **Multiply** the answer by the divisor and write it below the like terms of the dividend.
4. **Subtract** the bottom from the top.
5. **Bring down** the next term of the dividend.
6. **Repeat** steps 2–5 until reaching the last term of the dividend.
7. If the remainder is not zero, write it as a fraction using the divisor as the denominator.

4-03 Divide Polynomials (4.3)

$$\frac{y^4 + 2y^2 - y + 5}{y^2 - y + 1}$$

$$
\begin{array}{r}
y^2 + y + 2 \\
y^2 - y + 1 \overline{) y^4 + 0y^3 + 2y^2 - y + 5} \\
-\underline{y^4 - y^3 + y^2} \\
y^3 + y^2 - y \\
-\underline{y^3 - y^2 + y} \\
2y^2 - 2y + 5 \\
-\underline{2y^2 - 2y + 2} \\
3
\end{array}
$$

$$y^2 + y + 2 + \frac{3}{y^2 - y + 1}$$

4-03 Divide Polynomials (4.3)

$$\frac{x^3 + 4x^2 - 3x + 10}{x + 2}$$

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$$\begin{array}{r} x^2 + 2x - 7 \\ x + 2 \,\big)\, x^3 + 4x^2 - 3x + 10 \\ -\ \underline{x^3 + 2x^2} \qquad\qquad \\ 2x^2 - 3x \qquad \\ -\ \underline{2x^2 + 4x} \qquad \\ -7x + 10 \\ -\ \underline{-7x - 14} \\ 24 \end{array}$$

$$x^2 + 2x - 7 + \frac{24}{x + 2}$$

4-03 Divide Polynomials (4.3)

- **Synthetic Division**
  - Shortened form of long division for dividing by a binomial
  - Only when dividing by $(x - k)$

4-03 Divide Polynomials (4.3)

- **Synthetic Division**
- To divide a polynomial by $x - k$,

1. Write k for the divisor.
2. Write the coefficients of the dividend.
3. Bring the lead coefficient down.
4. Multiply the lead coefficient by k. Write the product in the next column.
5. Add the terms of the second column.
6. Multiply the result by k. Write the product in the next column.
7. Repeat steps 5 and 6 for the remaining columns.
8. Use the bottom numbers to write the quotient. The number in the last column is the remainder, the next number from the right has degree 0, the next number from the right has degree 1, and so on. The quotient is always one degree less than the dividend.

4-03 Divide Polynomials (4.3)

- Synthetic Division
  - $(-5x^5 - 21x^4 - 3x^3 + 4x^2 + 2x + 2) / (x + 4)$

Coefficients with placeholders

-4	-5	-21	-3	4	2	2
		20	4	-4	0	-8
	-5	-1	1	0	2	-6

$$-5x^4 - x^3 + x^2 + 2 + \frac{-6}{x+4}$$

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4-03 Divide Polynomials (4.3)

- $(y^5 + 32) \div (y + 2)$

-2	1	0	0	0	0	32
		-2	4	-8	16	-32
	1	-2	4	-8	16	0

- $y^4 - 2y^3 + 4y^2 - 8y + 16$

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Objectives:

- Evaluate a polynomial using the remainder theorem.
- List the possible rational zeros of a polynomial.
- Find the rational zeros of a polynomial.

4-04 Find Rational Zeros of Polynomial Functions (4.5)

4-04 Find Rational Zeros of Polynomial Functions (4.5)

- **The Remainder Theorem**
  - If a polynomial $f(x)$ is divided by $x - k$, then the remainder is the value $f(k)$.

- **Use the Remainder Theorem to Evaluate a Polynomial**
  - To evaluate polynomial $f(x)$ at $x = k$ using the Remainder Theorem,

1. Use synthetic division to divide the polynomial by $x - k$.
2. The remainder is the value $f(k)$.

4-04 Find Rational Zeros of Polynomial Functions (4.5)

- Use the remainder theorem to evaluate $f(x) = 3x^4 - 5x^3 + x - 14$ at $x = 2$.

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Use synthetic division with $x - 2$.

$f(2) = -4$

4-04 Find Rational Zeros of Polynomial Functions (4.5)

- **The Factor Theorem**
- According to the *Factor Theorem*, k is a zero of $f(x)$ if and only if $(x - k)$ is a factor of $f(x)$.
- **Use the Factor Theorem to Solve a Polynomial Equation**

To solve a polynomial equation given one factor using the factor theorem,

1. Use synthetic division to divide the polynomial by the given factor, $(x - k)$.
2. Confirm that the remainder is 0.
3. If the quotient is NOT a quadratic, repeat steps 1 and 2 with another factor using the quotient as the polynomial.
4. If the quotient IS a quadratic, factor the quadratic quotient if possible.
5. Set each factor equal to zero and solve for x.

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4-04 Find Rational Zeros of Polynomial Functions (4.5)

- Show that $x - 2$ is a factor of $x^3 + 7x^2 + 2x - 40$. Then find the remaining factors.

Use synthetic division with x − 2. Factor the quotient (depressed polynomial) to get the remaining factors.

All factors are $(x + 4)(x + 5)(x - 2)$

- Show that $x + 2$ and $x - 1$ are factors of $x^4 - 4x^3 - 3x^2 + 14x - 8$. Then find the remaining factors.

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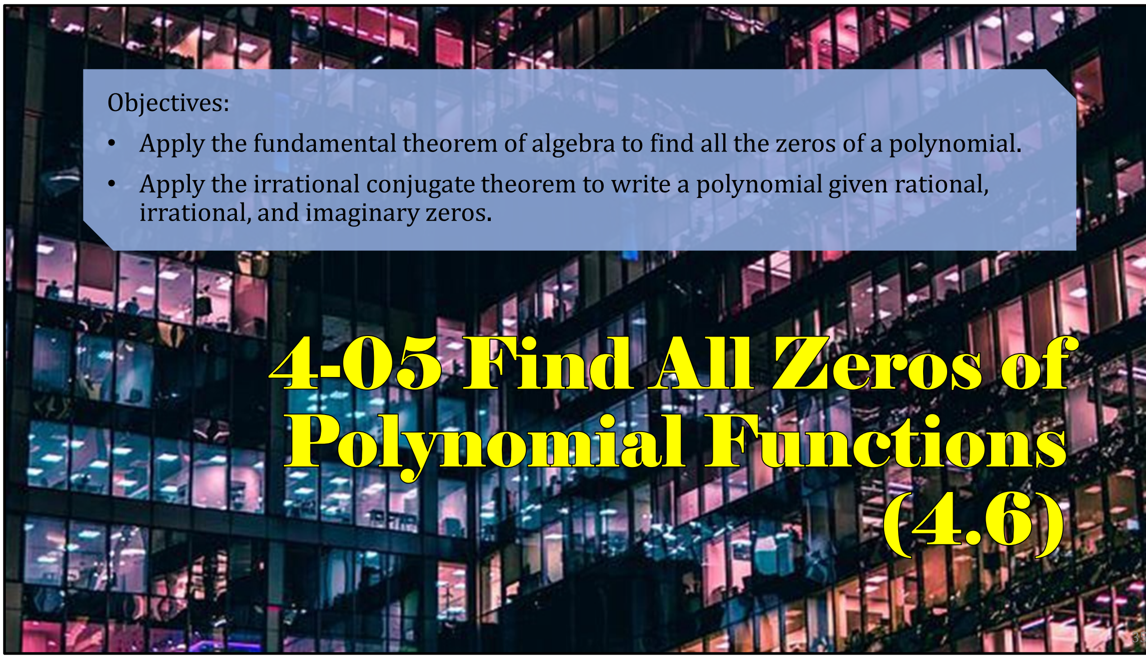
Use synthetic division with x + 2. The depressed polynomial is not a quadratic.
Use synthetic division with x − 1 with the depressed polynomial.
The new depressed polynomial is quadratic. Factor the quotient (depressed polynomial) to get the remaining factors.

All factors are $(x + 2)(x - 1)^2(x - 4)$


Objectives:
• Apply the fundamental theorem of algebra to find all the zeros of a polynomial.
• Apply the irrational conjugate theorem to write a polynomial given rational, irrational, and imaginary zeros.
4-05 Find All Zeros of Polynomial Functions (4.6)

4-04 Find Rational Zeros of Polynomial Functions (4.5)

- **Rational Zero Theorem**
  - Given a polynomial function, the rational zeros will be in the form of $\frac{p}{q}$ where p is a factor of the last (or constant) term and q is the factor of the leading coefficient.

4-04 Find Rational Zeros of Polynomial Functions (4.5)

- List all the possible rational zeros of
- $f(x) = 2x^3 + 2x^2 - 3x + 9$

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$p = \pm 1, \pm 3, \pm 9$
$q = \pm 1, \pm 2$
$p/q = \pm 1, \pm 1/2, \pm 3, \pm 3/2, \pm 9, \pm 9/2$

4-05 Find All Zeros of Polynomial Functions (4.6)

- **Use the Rational Zero Theorem and Synthetic Division to Find Zeros of a Polynomial**
- To find all the zeros of polynomial functions,

1. Use the Rational Zero Theorem to list all possible rational zeros of the function.
2. Use synthetic division to test a possible zero. If the remainder is 0, it is a zero. The *x*-intercepts on a graph are zeros, so a graph can help you choose which possible zero to test.
3. Repeat step two using the depressed polynomial with synthetic division. If possible, continue until the depressed polynomial is a quadratic.
4. Find the zeros of the quadratic function by factoring or the quadratic formula.

4-04 Find Rational Zeros of Polynomial Functions (4.5)

- Find all zeros of $f(x) = x^3 - 4x^2 - 2x + 20$

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List possible rational zeros;
$P = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
$q = \pm 1$
$p/q = \pm 1, \pm 2, \pm 4, \pm 5, \pm 10, \pm 20$
Use a graph to find an x-intercept that appears to be one of the rational zeros. (−2)
Use synthetic division to verify that it is a zero.
Since the remainder was zero −2 is a root and the depressed polynomial is $x^2 - 6x + 10$
Repeat the process on the depressed polynomial until you get a quadratic for the depressed polynomial then use the quadratic formula
$x = 3 \pm i, -2$

4-05 Find All Zeros of Polynomial Functions (4.6)

- **The Fundamental Theorem of Algebra**
- If $f(x)$ is a polynomial of degree $n > 0$, then $f(x)$ has at least one complex zero.

- A polynomial has the same number of zeros as its degree.

4-05 Find All Zeros of Polynomial Functions (4.6)

- How many solutions does $x^4 - 5x^3 + x - 5 = 0$ have? Find all the solutions.

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Four solutions
Factorable by grouping

$$x^4 - 5x^3 + x - 5 = 0$$
$$x^3(x - 5) + 1(x - 5) = 0$$
$$(x^3 + 1)(x - 5) = 0$$
$$(x + 1)(x^2 - x + 1)(x - 5) = 0$$
$$x + 1 = 0; x^2 - x + 1 = 0; x - 5 = 0$$
$$x = -1; x = \frac{1}{2} \pm \frac{\sqrt{3}}{2} i; x = 5$$

4-05 Find All Zeros of Polynomial Functions (4.6)

- Given a function, find the zeros of the function.

$$f(x) = x^4 - 6x^3 + 9x^2 + 6x - 10$$

Not factorable
Find *p*'s, *q*'s, and *p*/*q*
$p = \pm 1, \pm 2, \pm 5, \pm 10$
$q = \pm 1$
$p/q = \pm 1, \pm 2, \pm 5, \pm 10$
Use a graph to choose a *p*/*q* which is an *x*-intercept (1)
Use synthetic division to check to see if it is a factor (it is)
The depressed polynomial is not a quadratic, so use another *x*-intercept (−1)
Use synthetic division with the depressed polynomial to check to see if it is a factor (it is)
The depressed polynomial is a quadratic, so use the quadric formula to solve.
The zeros are 1, −1, $3 \pm i$

4-05 Find All Zeros of Polynomial Functions (4.6)

- **Complex Conjugate Theorem**
  - If the complex number $a + bi$ is a zero, then $a - bi$ is also a zero.
  - Complex zeros come in pairs
- **Irrational Conjugate Theorem**
  - If $a + \sqrt{b}$ is a zero, then so is $a - \sqrt{b}$